Two-Component LDV Measurements in a Stirred Tank

Previous work has shown that in a standard stirred tank, the three-dimensional velocity field in the discharge flow of the stirrer is characterized by a strong deviation from homogeneous isotropic turbulence. By means of a two-component laser Doppler velocimeter (2D-LDV), the $\overline{v_r'v_\theta'}$ Reynolds stresses are measured in a 6.3-dm^3 standard tank stirred by a six-flat-blade Rushton turbine. Obviously, these stresses must be known to better understand the turbulent mixing in such a configuration. Moreover, the 2D-LDV appears to be an efficient tool for characterizing, from an energy viewpoint, the type of agitator used and, in principle, is confirmed to be useful to accurately obtain the length scales of turbulence which are fundamental parameters for the study of the micromixing processes.

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Introduction

A fine knowledge of the hydrodynamics and of the turbulence features in stirred reactors is primordial for the development of models on turbulent mixing in such vessels. The laser Doppler velocimetry has asserted itself against the thermal anemometry as a mean of investigating the flow field (Costes and Couderc, 1984). In spite of the numerous work on the subject, accurate and sufficiently reliable data are still lacking. For example, information can be rarely found at a given measuring point about the three components of the velocity and their fluctuations (Barthole et al., 1982; Mahouast et al., 1987), and their simultaneous recording is again scarcer. Most works (Ito et al., 1975; Günkel and Weber, 1975; Costes and Couderc, 1982; Mahouast et al., 1987, 1988) demonstrate the strong inhomogeneity and anisotropy of the turbulence in the discharge flow of the turbine. The first attempt to measure Reynolds stresses was made by Ito et al. (1975). Drbohlav et al. (1978) published some profiles of the axial-radial Reynolds stresses in a discharge flow. Ito et al. (1975) used an electrochemical intrusive technique (spherical probe) based on mass transfer, allowing simultaneous measurements of three velocity components. In fact, their measurements cannot be considered as being carried out exactly at the same point of the space. Elsewhere, the frequency response of such a probe is not well known.

In the present work, a two-component laser Doppler velocimeter has been brought into operation to measure the Reynolds stresses in a 6.3-dm^3 standard tank filled with water and stirred by a Rushton turbine. The stirring Reynolds number had a magnitude of 35,000. The influence of the stirring speed N of the turbine on these stresses has also been studied (Mahouast, 1988).

The aim of the present work is to give new and reliable data for a better analysis of the turbulent mixing in a stirred vessel, from experimental results obtained by a 2D LDV which has been running well in spite of the geometrical constraints of the experimental apparatus.

Measurements show that the Reynolds stresses $\overline{v_r'v_\theta'}$, whose values are nearly zero in the circulating zones, cannot be ignored in the discharge flow, since they attain up to 0.4 times the values of the variances. Besides, they show intense radial and axial gradients with sign changes. So, the concept of an eddy viscosity of the Boussinesq type cannot apply. The discharge flow works like a radial swirling pulsed jet. In addition, the existence of organized structures attached to the blade is revealed by the periodical auto- and cross-correlation curves of the velocity fluctuations. This probably does not favor mixing.

It is shown that the knowledge of simple and double correlations of the velocity fluctuations over a control surface around the stirrer is essential to determine the power transmitted to the fluid by the agitator and the part of this power resulting in the fluctuations (25% for a Rushton turbine). Other than the determination of the power number, the 2D LDV appears to be an efficient technique for measuring space-time correlations.

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Without ambiguity, they allow to attain the turbulence length scales when referring to the complexity of the flow in a stirred tank, such as three-dimensional, periodic component in the flow, strong shear rates, and high intensity of the turbulence (validity of the Taylor's assumption), nonhomogeneity and anisotropy.

Apparatus and Experimental Technique

Mixing vessel

The standard reactor of the Holland and Chapman type (1966) comprises a vessel with four baffles and a stirrer. These parts are made of a transparent material (PMMA). Their dimensions and the position of the agitator are determined from the knowledge of the tank diameter (see Figure 1).

The cylindrical tank, whose base is circular and flat, is tooled in a cube. The four baffles, vertical and flush with the wall, make an integral assembly which can be angularly positioned around the vessel axis, making it easier to measure. The bottom is provided by an optical window for the passage of laser beams reflected by a plane mirror fitted under the reactor (Figure 1). The vessel is filled with water at a 25°C temperature. On the free surface of the liquid, a circular optical window can be displaced through articulations and allows a good focusing of the photomultiplier optics on the measuring point.

The agitator (Rushton turbine composed of a disc with six rectangular blades) is coupled with a variable speed motor set above the tank on a frame through silent blocks.

The all assembly (tank, frame + motor) lays on a 3D displacement table (precision:0.025 mm) used for positioning the measuring volume.

Cylindrical (r, θ, z) coordinates are used to locate the measuring point. The θ angle is oriented in the direction of the rotation of the turbine, the origin corresponding to a baffle. The origin of the vertical z axis is the center of the turbine disc.

Two-Component Laser-Doppler velocimeter with frequency shifting

The basical background of the laser Doppler velocimetry will not be discussed here, since it has been detailed in many books (see, for instance, Goldstein, 1983).

The two-component LDV used to measure the Reynolds stresses in this work will be now shortly described.

The light source is an Argon laser. The power of the emitted beam (blue-green) can be regulated up to 5 W. This beam is splitted by a modular optical system into three parallel beams equidistant to the optical axis: a blue one B (488 nm), a green one G (515 nm), and the blue-green (BG). B and G beams are 40 MHz in frequency shifted with respect to the blue and green rays of the BG beam, by aids of a Bragg cell, in order to determine measured velocity components. The plane of B and BG beams is perpendicular to the plane of G and BG beams. At the focus of a converging lens having the same optical axis as the system, these three beams give, at their intersection, two perpendicular networks of blue fringes and green fringes. Figure 2 shows the projection of the measuring volume on a screen. Owing to the frequency shifting, each network moves at a constant velocity perpendicular to the fringes. Receipt of forward

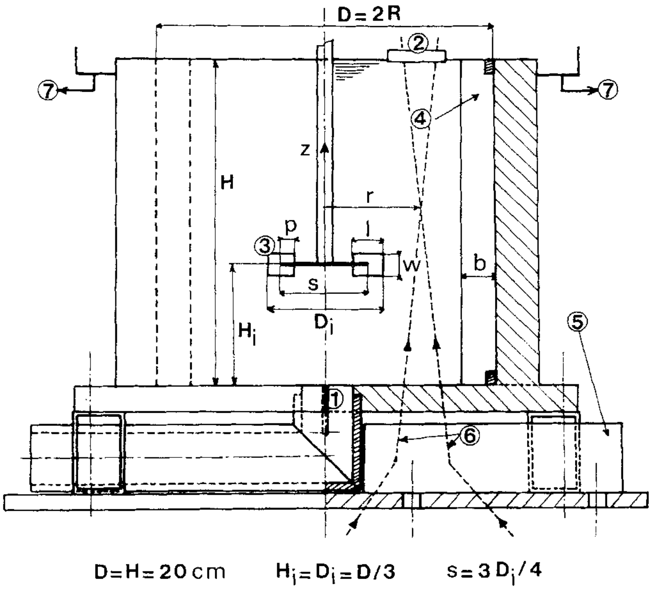

Figure 1. Mixing vessel and laser beams.

1. Injector
2. Optical window
3. Rushton turbine
4. Baffle
5. Mirror
6. Laser beams
7. Water evacuation

In the present work, as the reactor runs in batch mode, the supplies of fresh water and salt water shown on the figure are not used. Only two laser beams, not three, are drawn.

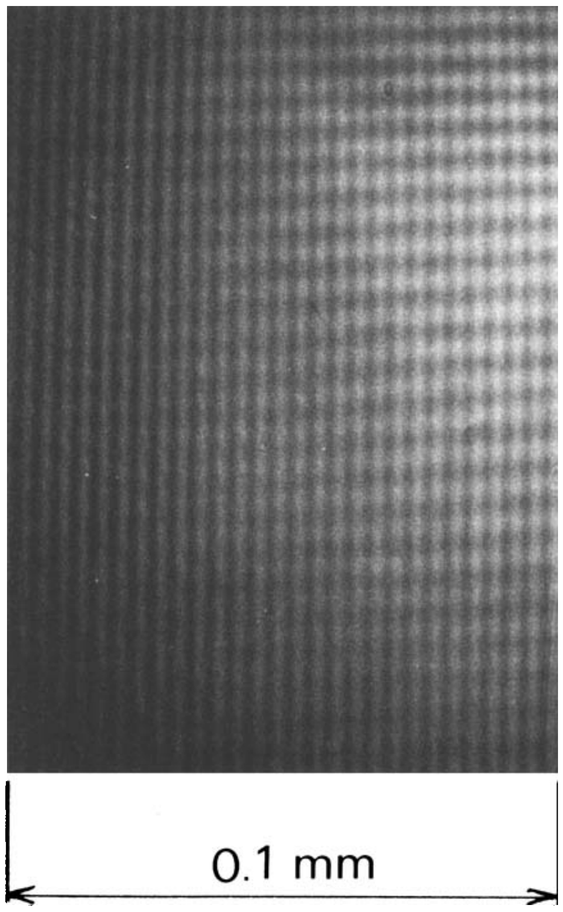

Figure 2. Perpendicular networks of the blue and green fringes in the laser measuring volume (2D-LDV).

scattered lights (Doppler signals) emitted by particles and moving through this measuring volume is made by a photomultiplier placed above the tank. The receiving optics comprise a color separator and two interference narrow band filters (blue and green). Then, each signal is processed through a frequency mixing unit and through a frequency tracker. In the latter case, an analog output is available.

The signals are then numerically converted and processed to obtain any usual statistical function and to analyze the spectra. The frequency sampling was 2 kHz. At a given measuring point, 60 consecutive samples of 2,048 points were recorded on a disk, simultaneously for the two velocity components. Processing time was most time deferred to compute the following quantities and functions such as: mean values of the velocities, root-mean-square values of the fluctuations, Reynolds stresses, coefficients of auto and cross correlations, spectra and cross spectra, macro- and microscales of the turbulence.

Before carrying out the measurements in the vessel, the LDV has been proofed (value and sign of the velocity) in a known flow field: the solid rotation of water in a cylinder made of PMMA. So, accuracy on the velocity has been found to be lower than 1%.

Demineralized water was used in the mixing tank to reduce the presence of natural particles, and then a progressive seeding of the liquid by silicon oil particles (1–3 μm) was made until best-quality Doppler signals were obtained.

Reynolds Stresses for Calculating Power Transmitted to Fluid by Stirrer

From a theoretical viewpoint, the Reynolds stresses are obviously important, since a closure of turbulent motion equations is needed to solve the problem.

The purpose here is to show the part played by these stresses in the power received by the fluid passing through the stirrer. Cutter (1966) and, more recently, Placek et al. (1986) discussed the dissipation problem of the impeller power into average velocity and velocity fluctuations. Unfortunately, the previous paper did not rely on extensive experimental data, since they are not available; Placek et al. (1986) based their model on numerical simulation using only experimental mean velocities.

As considered by several authors (Günkel and Weber, 1975), let Σ be a control surface around the agitator (see Figure 3). The

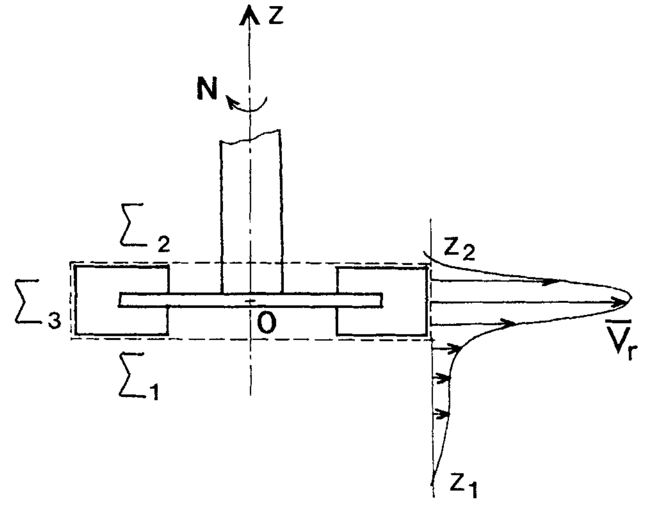

Figure 3. Control surface Σ around the stirrer.

balance of total mechanical energy rates across Σ gives the global power received by the fluid crossing this surface:

$$\boldsymbol{P} = \overline{\int_\Sigma \left(P + \frac{1}{2}\rho V^2\right) \vec{V} \cdot \vec{n}\, d\sigma}, \tag{1}$$

where

$\vec{n}$ = normal unity vector external to Σ

$\vec{V}(V_r, V_\theta, V_z)$ = instantaneous local velocity

P = instantaneous driving pressure

Using the Reynolds decomposition $V_i = \overline{V_i} + v_i'$, $P = \overline{P} + p'$, and neglecting the triple correlations, the following powers result:

On Σ_1:

$$\boldsymbol{P}_1 = \int_0^{D_i/2} \left\{ \left[\overline{P} + \frac{1}{2}\rho(\overline{V_r}^2 + \overline{V_\theta}^2 + \overline{V_z}^2)\right]\overline{V_z} + \frac{1}{2}\rho(\overline{v_r'^2} + \overline{v_\theta'^2} + 3\overline{v_z'^2})\overline{V_z} + \rho\overline{V_r}\cdot\overline{v_r'v_\theta'} + \rho\overline{V_\theta}\cdot\overline{v_\theta'v_z'} + \overline{p'v_z'} \right\} 2\pi r\, dr \tag{2}$$

On Σ_2: an expression $\boldsymbol{P}_2$ similar to $\boldsymbol{P}_1$

On Σ_3:

$$\boldsymbol{P}_3 = \int_{-w/2}^{+w/2} \left\{ \left[\overline{P} + \frac{1}{2}\rho(\overline{V_r}^2 + \overline{V_\theta}^2 + \overline{V_z}^2)\right]\overline{V_r} + \frac{1}{2}\rho(3\overline{v_r'^2} + \overline{v_\theta'^2} + \overline{v_z'^2})\overline{V_r} + \rho\overline{V_\theta}\cdot\overline{v_r'v_\theta'} + \rho\overline{V_z}\cdot\overline{v_\theta'v_z'} + \overline{p'v_r'} \right\} \pi D_i\, dz \tag{3}$$

with

$$\boldsymbol{P} = \boldsymbol{P}_3 - (\boldsymbol{P}_1 + \boldsymbol{P}_2) \tag{4}$$

In Eq. 4, there is a part which depends on fluctuations as well as variances, Reynolds stresses, $\overline{v_r'v_\theta'}$, $\overline{v_r'v_z'}$, $\overline{v_\theta'v_z'}$, and pressure-velocity correlations, $\overline{p'v_z'}$ and $\overline{p'v_r'}$, are present. Before trying to simplify the problem, these terms must be known.

Now, $\boldsymbol{P}$ can be computed from the torque of external forces applied to the control volume. The system of these forces is equivalent to that of momentum rates through the surface. Therefore, the knowledge of the velocity repartition along Σ suffices to determine the torque exerted by the stirrer on the fluid. This torque is:

$$\vec{M} = \int_\Sigma (\vec{r} \wedge \rho\vec{V})\vec{V}\cdot\vec{n}\, d\sigma$$

with respect to the z axis.

Because of the axisymmetry of the flow, the relation becomes:

$$\vec{M} = \vec{z} \int_{\Sigma_1} \rho r V_\theta \vec{V} \cdot \vec{n} \, d\sigma$$

where $\vec{z}$ is the unit vector in the z direction and algebraically:

$$M_z = - \int_{\Sigma_1} \rho r V_\theta V_z \, d\sigma + \int_{\Sigma_2} \rho r V_\theta V_z \, d\sigma + \int_{\Sigma_3} \rho r V_\theta V_r \, d\sigma \quad (7)$$

After Reynolds' decomposition and time averaging:

$$\overline{M_z} = - \int_0^{D_i/2} \rho(\overline{V_\theta}\,\overline{V_z} + \overline{v_\theta' v_z'})\, 2\pi r^2 \, dr + \int_0^{D_i/2} \cdot \rho(\overline{V_\theta}\,\overline{V_z} + \overline{v_\theta' v_z'})\, 2\pi r^2 \, dr + \int_{-w/2}^{w/2} \frac{\rho}{2} (\overline{V_r}\,\overline{V_\theta} + \overline{v_r' v_\theta'}) \pi D_i^2 \, dz \quad (8)$$

Equation 8 shows a term which depends on the mean flow and a term which is a function of the Reynolds stresses along Σ. Finally, the power of the turbine is obtained by multiplying $\overline{M_z}$ by the angular velocity of impeller blades:

$$\mathbf{P} = \overline{M_z} \cdot 2\pi N \quad (9)$$

In the expression (Eq. 4) of $\boldsymbol{P}$, let $\boldsymbol{P'}$ be the part which depends on fluctuations. $\boldsymbol{P'/P}$ can be defined as a parameter characterizing the stirrer on an energetic viewpoint. If, indeed, blades have such a geometry that the power of fluctuations is not increased at the Σ crossing, $\boldsymbol{P'/P}$ will be zero. On the other hand, this parameter is less than 1. At first sight, it seems logical that mixing in a stirred vessel would be improved by the use of an agitator having a large $\boldsymbol{P'/P}$ ratio. In the next part, $\boldsymbol{P}$ and $\boldsymbol{P'}$ will be calculated, and the Reynolds stresses contribution will be estimated.

Experimental Results and Discussion

Cross correlations and Reynolds stresses

The experimental results concern the $\overline{v_r' v_\theta'}$ term, without decomposing each fluctuation in the sum of a periodic component v_{ip}' and a turbulent component v_{it}', because this would be impracticable in the present case. Periodic fluctuations have been studied by several authors (Van der Molen and Van Maanen, 1978; Günkel and Weber, 1975) and more recently by Mahouast and Cognet (1987) who gave a physical model of these fluctuations in the discharge flow.

The present experiments have been done at a sufficiently high Reynolds number so that $\overline{v_r' v_\theta'}$ is proportional to N^2 (Mahouast, 1988).

Exploration along the Discharge Flow. The curve of the cross-correlation coefficient of the v_r' and v_θ' fluctuations is periodic up to a $x = 0.2$ distance from the blade (Figures 4 and 5). The time lag τ_o between these fluctuations decreases when x increases. In Figure 6, the simultaneous recording of v_r' and v_θ' signals, a difference in phase can be observed. Cross spectra, $S_{r\theta}$, in Figure 7 show that these fluctuations have common characteristics of frequencies f^*, which is the frequency of the blade passage, and multiples of f^* near the blades, then only f^*, whereas low frequencies become more important in the real part of $S_{r\theta}$, and finally a little farther away from the blades the cross spectrum becomes real and continuous.

It can be shown rather simply that the existence of an imaginary part in $S_{r\theta}$ is the consequence of phase shifts between the constitutive terms of the Fourier Transforms of the two signals.

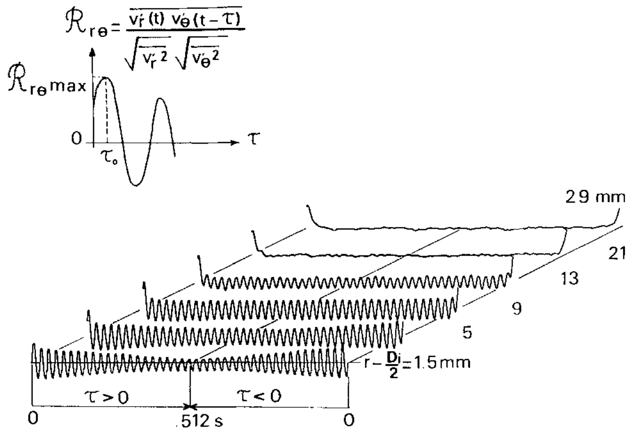

Figure 4. Cross-correlation curves of v_r' and v_θ' fluctuations in the discharge flow: ($z = 0$, $\theta = 45°$); $N = 7 s^{-1}$.

Let a_n, b_n and a_n', b_n' be Fourier coefficients and f_e the sampling frequency. Then:

$$S_{r\theta}(nf_e) = (a_n + ib_n)(a_n' - ib_n')/4 \quad (10)$$

with

$$tg\phi_n = -b_n/a_n \quad (11)$$

$$tg\phi_n' = -b_n'/a_n' \quad (12)$$

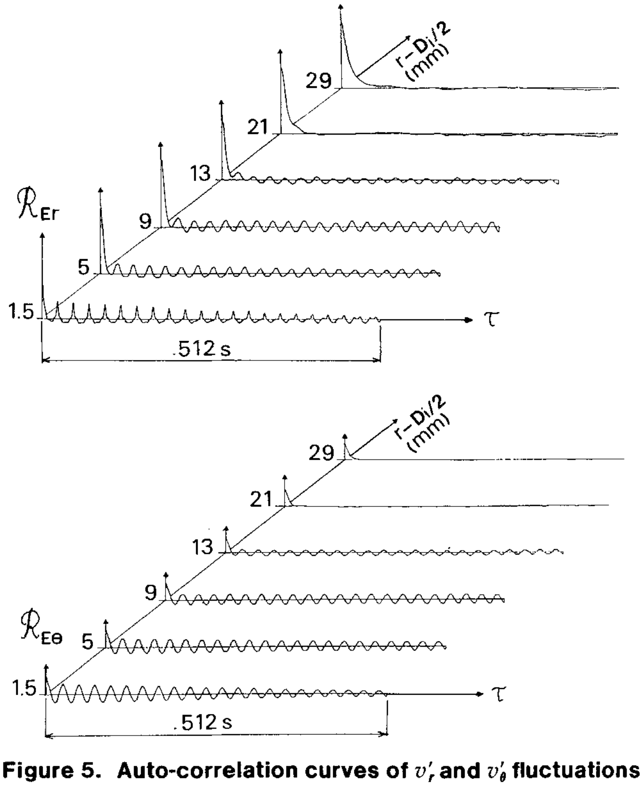

Figure 5. Auto-correlation curves of v_r' and v_θ' fluctuations in the discharge flow: ($z = 0$, $\theta = 45°$); $N = 7 s^{-1}$.

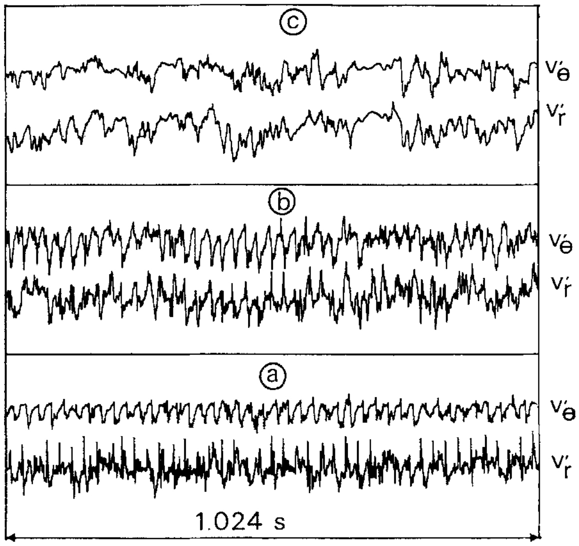

Figure 6. Simultaneous signals of the v'_r and v'_θ fluctuations in the discharge flow ($z = 0$, $\theta = 45°$).
$N = 7\ s^{-1}$; $r - D_i/2 = 1.5$ mm for **a**; 5 mm for **b**; 29 mm for **c**.

Then:

$$S_{r\theta} = \frac{a_n a'_n}{4 \cos \phi_n \cdot \cos \phi'_n} \left[\cos\left(\phi'_n - \phi_n\right) + i \sin\left(\phi'_n - \phi_n\right)\right] \quad (13)$$

The absence of an imaginary part in the cross spectrum implies that $\phi'_n = \phi_n$ modulo π for each n value. So, in the Fourier decomposition, the nth components of the two signals are in phase or in phase opposition. In any case, there must exist some resemblance between the two signals. At 29 mm from the blades, the cross spectrum is entirely real, and Figure 6 shows that the signals present an astonishing likeness. At this distance where no periodic contribution subsists in the fluctuations, the maximal correlation coefficient $R_{r\theta}$ has a value of 0.34, which is large enough and the Reynolds stress $\overline{v'_r v'_\theta}$ remains still significant with respect to the variances in $\overline{v'^2_r}$, $\overline{v'^2_\theta}$ (Figures 8 and 9). At $x = 0.12$ (i.e., $r - D_i/2 = 8$ mm), Figures 8 to 10, a maximum for the cross-correlation coefficient, the Reynolds stress, and the turbulent kinetic energy are remarkably observed. Moreover, this distance also corresponds to a rapid increase of the length macroscale L (Figure 11). These consentrations can be linked to the existence of organized structures around the stirrer, since the decay of turbulent vortices in the stream streaking from the body of turbine impeller should lead to decrease of the turbulence macroscale, L.

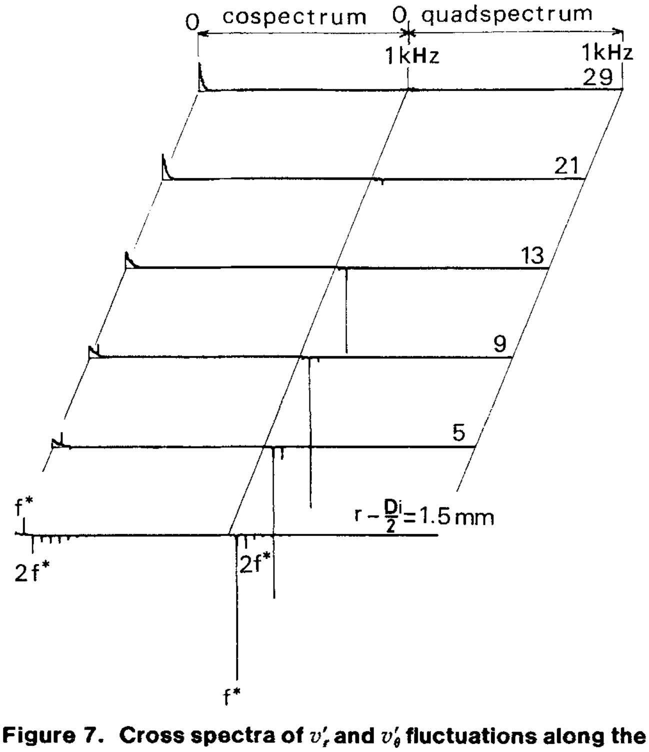

Figure 7. Cross spectra of v'_r and v'_θ fluctuations along the discharge flow ($z = 0$, $\theta = 45°$): $N = 7\ s^{-1}$.

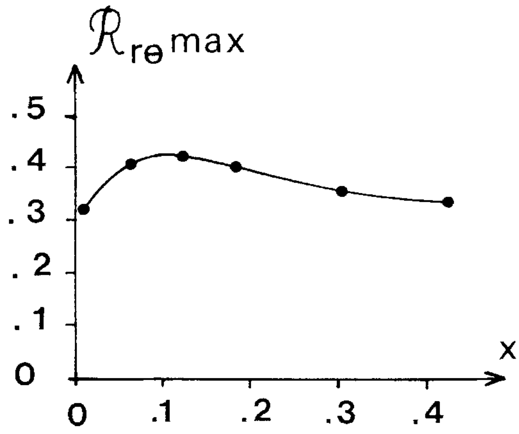

Figure 8. Evolution of the maximal cross-correlation coefficient between v'_r and v'_θ in the discharge flow.

Existence of Organized Structures. In the discharge flow, when considering the autocorrelation curves (Figure 5) and the cross-correlation curves (Figure 4), it can be seen that they are

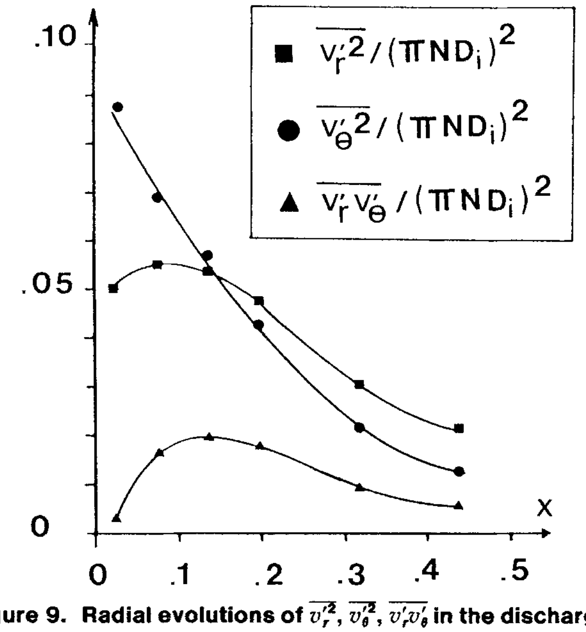

Figure 9. Radial evolutions of $\overline{v'^2_r}$, $\overline{v'^2_\theta}$, $\overline{v'_r v'_\theta}$ in the discharge flow ($z = 0$, $\theta = 45°$).

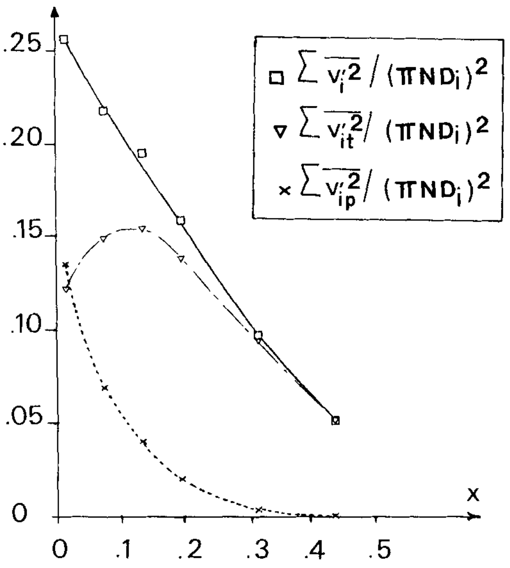

Figure 10. Kinetic energy of fluctuations in the discharge flow ($z = 0$, $\theta = 45°$).
□ global fluctuation
▽ turbulent component
x periodic component

quite sinusoidal. Thus, they show that, after one revolution of the turbine, the auto- and cross-correlation coefficients keep the same values. Moreover, no significant loss of correlation can be observed between the maxima of the curves at successive time intervals of $1/N$. This clearly confirms that the same turbulent structure is present again after an integral number of revolutions of the turbine and thus that well-organized structures remain attached to the blades.

This result can probably be connected to the presence of the well-known trailing vortices behind the blade (Van't Riet and Smith, 1974), Figure 12. Elsewhere, this result can be brought together with the existence of stable gas cavities behind the blades in gas-liquid flow which has been studied by several authors (Bruijn et al., 1974; Nienow and Wisdom, 1974; Nienow et al., 1985).

From the forementioned curves, $x = 0.12$ appears to be the

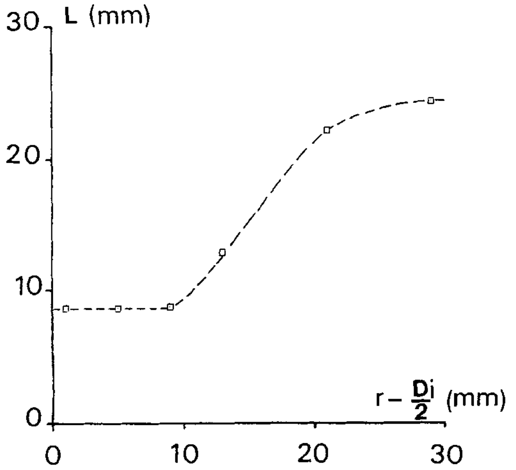

Figure 11. Evolution of the length macroscale L along the discharge flow ($z = 0$, $\theta = 45°$): $N = 7\ s^{-1}$.

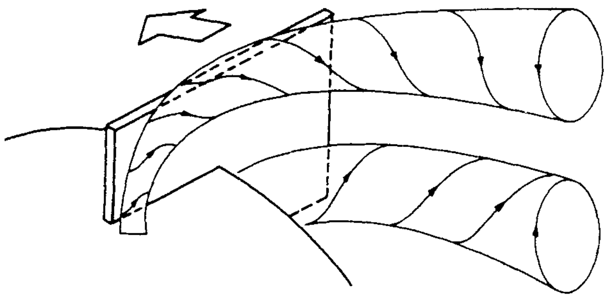

Figure 12. Trailing vortices behind the blade (Van't Reit and Smith, 1974).

point of separation, in the discharge flow, between the trailing vortices and the surrounding fluid. The existence of these organized structures rotating with the impeller does probably not improve mixing in the tank because this can appear as some kind of segregation. It would be better that these structures disappear or break away from the blades to be convected within the rest of the tank where isotropic mixing will ensure concentration homogeneity up to the molecular scale.

Reynolds Stresses along Two Vertical Axes. Figure 13 shows the $\overline{v_r'v_\theta'}$ Reynolds stress and the variances of v_r' and v_θ' along two vertical axes located at 5 mm and 45 mm from the blades.

From approximately $z = -3w/2$ to $z = 3w/2$, along the 5 mm axis, it can be seen that the Reynolds stresses, compared to the variances, are significant. They also show sign changes and

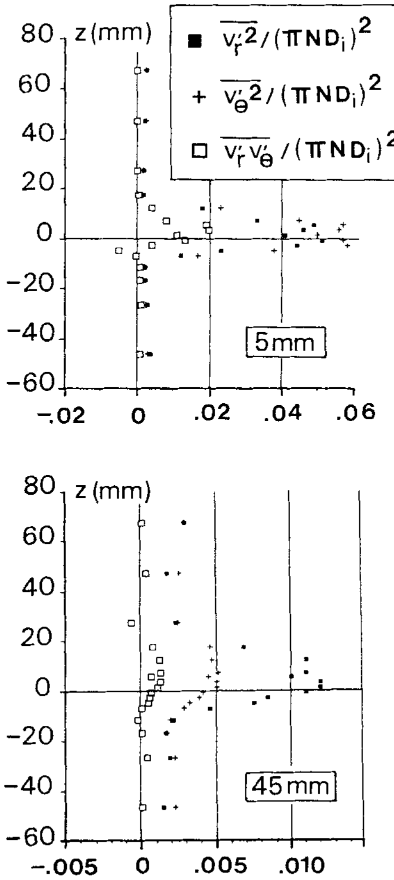

Figure 13. Axial evolutions of $\overline{v_r'^2}$, $\overline{v_\theta'^2}$, $\overline{v_r'v_\theta'}$ at 5 mm and 45 mm from blades ($\theta = 45°$).

intense gradients. The profiles appear to be similar to what can be found in a radial swirling jet (Muhé and Combarnous, 1986) or in an axially symmetrical tangential jet (De Souza and Pike, 1972), although these are not considered as submitted to a periodic excitation as the one due to the blades in the present work.

The question arises if the observed large values are linked or not to the correlation of the periodic fluctuations v'_{rp} and $v'_{\theta p}$. The answer is probably not since, at a 45-mm distance from the blade where the periodic phenomenon has vanished, the profile of the Reynolds stress keeps the same shape as at 5 mm and the values are still significant (Figures 9 and 13).

In the region $|z| > 3w/2$ and $x > 0$, the Reynolds stresses are practically zero. This suggests an isotropic turbulence in these zones, knowing elsewhere that the variances of three fluctuations are nearly equal (Mahouast et al., 1987).

Power considerations

Power Number. As in relations 2–4, when the driving pressure $\overline{P}$ is unknown, the relation 8 of the torque will be used. In that relation, the contribution of the Reynolds term $\overline{v'_\theta v'_z}$ will not be considered as it has not been measured. Besides, it is probably negligible with respect to $\overline{V}_\theta \overline{V}_z$.

The surfaces Σ_1 and Σ_2 (Figure 3) used for the calculus correspond to levels z_1 and z_2, where the velocity profiles have been measured by a 1D LDV (Mahouast et al., 1987). z_1 and z_2 are such that $\overline{V}_r(z_1) = \overline{V}_r(z_2) = 0$. So, in expression 8, the three integrals divided by $\rho N^2 D_i^5$ give 0.084, 0.063, and 0.567, respectively, in which $\overline{v'_r v'_\theta}$ contributes to the value 0.050. Then the torque coefficient is worth:

$$\overline{M_z}/\rho N^2 D_i^5 = 0.714 \tag{14}$$

and the power number

$$N_p = \overline{M_z} 2\pi N/\rho N^3 D_i^5 = 4.50 \tag{15}$$

These results agree well with the values of Nagata (1975), $N_p = 5$ for $R_e > 10^4$. It would be more accurate if 2D-LDV measurements were carried out along surfaces Σ_1 and Σ_2 as close as possible to the impeller. An interesting result is that the Reynolds stress $\overline{v'_r v'_\theta}$ contributes, by nearly 10%, to the power term P_3 calculated along the outlet surface Σ_3. It must be kept in mind that the contribution of the two other Reynolds stresses has been omitted so that the part played by the turbulent shear stresses may be more important.

Power Transferred to Fluctuations. In the relations 2 and 4, the Reynolds stresses $\overline{v'_r v'_z}$ and $\overline{v'_\theta v'_z}$ have not been measured. The pressure-velocity correlation terms $\overline{p'v'_z}$ and $\overline{p'v'_r}$ are not known and are difficult to measure.

In spite of our previous remarks, the order of magnitude of the terms $\overline{v'_r v'_z}$ and $\overline{v'_\theta v'_z}$ appears to be smaller than $\overline{v'_r v'_\theta}$ Ito et al. (1975). For convenience, the pressure-velocity correlation terms will be omitted. Then,

$$P' = \frac{1}{2}\rho \int_{-w/2}^{w/2} [(3\overline{v_r'^2} + \overline{v_\theta'^2} + \overline{v_z'^2})\overline{V}_r + 2\overline{V}_\theta \cdot \overline{v'_r v'_\theta}]\pi D_i \, dz$$
$$- \frac{1}{2}\rho \int_{\Sigma_1,\Sigma_2} (\overline{v_r'^2} + \overline{v_\theta'^2} + 3\overline{v_z'^2})\overline{V}_z \, d\sigma \tag{16}$$

According to the present measurements, the calculations are made for surfaces Σ_1 and Σ_2 relative to the z_1 and z_2 levels where $\overline{V}_r = 0$. Therefore, in Eq. 16,

- First integral divided by $\rho N^3 D_i^5$ = 1.25
- Reynolds stress contributing = 0.14
- Second integral divided by $\rho N^3 D_i^5$ = 0.03

Taking the value $N_p = 5$ for the power number of the Rushton turbine ($Re > 10^4$), it follows:

$$P'/P = 0.25 \tag{17}$$

This result shows that 25% of the power transmitted by the turbine to the fluid flowing through goes to the fluctuations. So, the Rushton turbine, apart from the fact that it maintains the liquid circulation in the vessel, also creates in the crossing fluid strong fluctuations which in turn improve the mixing process. Several types of agitators (e.g., propeller), having a rather good profile, generate mainly the liquid movement in the tank, and fluctuations exist only by the turbulent nature of the flow. Extreme cases can be found in turbomachinery in which turbulence is a handicap. In such installations, the best profiles of the blade shape are searched to minimize the energy losses, because the latter increases rapidly with the intensity of turbulence. Fort (1986) evaluated the fraction of impeller power transmitted to the rest of the tank for a stirred tank with a six 45°-pitched-blade impeller between 0.6 and 0.7, depending on the stirrer/tank diameter ratio.

Coming back to the fluctuations created by a Rushton turbine, the energetic part, which is carried by the periodic component just at the outlet of the turbine, is found to be nearly equivalent to the turbulent part.

Assumptions for mixing models in a standard stirred tank

In the discharge flow, it has been seen that Reynolds stresses cannot be neglected. So, theoretical results of homogeneous isotropic turbulence are not applicable.

As the turbulence is very intense in the vessel and as the gradients of the mean velocity $\overline{V}$ are strong, the Taylor's assumption which permits the passage from a time scale to a length scale by multiplying $\overline{V}$ does not seem suitable. Nevertheless, many results are given using isotropic turbulence and the Taylor's hypothesis. So, the dimensionless dissipation rate ϵ^* of turbulent kinetic energy at the same location varies within a ratio from 1 to 70 according to the literature (Laufhütte and Mersmann, 1987; Patterson and Wu, 1985). These discrepancies cannot be explained solely by different measuring techniques if the two preceding points are not considered.

In the region $|z| > 3w/2$, $x > 0$, the turbulence turns to homogeneity and isotropy.

Previous work (Mahouast et al., 1987) tends to confirm that, in the rotary zone around the turbine shaft, the turbulence deviates from isotropy. The flow in this region is like an axial swirling jet.

The vertical and radial gradients of the $\overline{v'_r v'_\theta}$ stress change sign. Then, it can be seen that the concept of an eddy viscosity ν_t

such that:

$$\overline{v_r'v_\theta'} = -\nu_t\left(\frac{\partial\overline{V_r}}{r\partial\theta} + \frac{\partial\overline{V_\theta}}{\partial r}\right) \tag{18}$$

does not apply in the discharge flow, because $(\partial\overline{V_r})/(r\partial\theta)$ is zero (axisymmetric flow) and $(\partial\overline{V_\theta})/(\partial r)$ is always negative.

Local isotropy of small scales exists, however, even if large scales are largely anisotropic in the discharge stream (Patterson and Wu, 1985).

In consequence, according to the three forelimited zones (points 1, 3, 4), it seems that a combination of different models of turbulence must be used to predict the turbulent mixing in the entire vessel.

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Notation

a_n, a_n' = cosine Fourier coefficients
b_n, b_n' = sine Fourier coefficients
D = vessel diameter
$D_i = D/3$ = turbine diameter
$f^* = 6N$ = frequency of blade passage
f_e = sampling frequency
$H = D$ = vessel height
$H_i = D_i$ = turbine clearance above the base
$\underline{L} = (\Lambda_r\Lambda_\theta\Lambda_z)^{1/3}$ = length macroscale of turbulence
$\vec{M}(M_r, M_\theta, M_z)$ = torque exerted by the turbine on the fluid
N = stirring speed
$\vec{n}$ = external unity normal vector
$N_p = P/\rho N^3 D_i^5$ = power number
P = driving pressure
p' = pressure fluctuation
$\mathbf{P}$ = power input
$\mathbf{P}'$ = power transmitted to fluctuations at Σ crossing
$\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$ = total mechanical energy rates through Σ_1, Σ_2, Σ_3 surfaces
$\overline{q^2} = \Sigma\overline{v_i'^2}$ = twice the kinetic energy of the fluctuations
$R = D/2$ = vessel radius
$R_e = ND_i^2/\nu$ = Reynolds number
r = radial coordinate
$R_{Ei}(\tau)$ = autocorrelation coefficient of v_i' fluctuation
$R_{r\theta}(\tau)$ = cross-correlation coefficient of v_r' and v_θ' fluctuations
$S_{r\theta}$ = cross spectrum of v_r' and v_θ' fluctuations
V = water volume in the tank
$\vec{V}(V_r, V_\theta, V_z)$ = instantaneous velocity
V_i = instantaneous i component of velocity
$v_i' = v_{ip}' + v_{it}'$ = global fluctuation of i velocity component
v_{ip}' = periodic component of i fluctuation
v_{it}' = turbulent component of i fluctuation
w = blade height
$x = (r - D_i/2)/D_i$ = dimensionless coordinate
z = axial coordinate
z_1, z_2 = particular levels

Greek letters

ρ = specific mass of the fluid
ν = kinematic viscosity
θ = angular coordinate
ϵ = dissipation rate of turbulent kinetic energy
$\epsilon^* = \epsilon/(\mathbf{P}/\rho V)$ = dimensionless dissipation rate of turbulent kinetic energy
τ = time (abscissa of correlation curves)
τ_o = time of the maximum of cross correlation
Λ_i = length macroscale in i direction
$\Sigma = \Sigma_1 U \Sigma_2 U \Sigma_3$ = control surface around stirrer
ϕ_n, ϕ_n' = phases

Subscripts

i = index for r, θ, z
k, j = integers (1, 2, or 3)
n = integer
p = periodic
t = turbulent
1, 2, 3 = indexes for surfaces of control volume

Superscript

$\overline{}$ = time averaging

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